

INTERSECTIONS OF FRÉCHET SPACES AND (LB)-SPACES

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ABSTRACT. This article presents results about the class of locally convex spaces which are defined as the intersection $E \cap F$ of a Fréchet space F and a countable inductive limit of Banach spaces E . This class appears naturally in analytic applications to linear partial differential operators. The intersection has two natural topologies, the intersection topology and an inductive limit topology. The first one is easier to describe and the second one has good locally convex properties. The coincidence of these topologies and its consequences for the spaces $E \cap F$ and $E + F$ are investigated.

1. Introduction. The aim of this paper is to investigate spaces $E \cap F$ which are the intersection of a Fréchet space F and an (LB)-space E . They appear in several parts of analysis whenever the space F is determined by countably many necessary, e.g., differentiability of integrability, conditions and E is the dual of such a space, in particular E is defined by a countable sequence of bounded sets which may also be determined by concrete estimates. Two natural topologies can be defined on $E \cap F$: the intersection topology, which has semi-norms easy to describe and which permits direct estimates, and a finer inductive limit topology which is defined in a natural way and which has good locally convex properties, e.g., $E \cap F$ with this topology is a barreled space. It is important to know when these two topologies coincide. It turns out that the locally convex properties of $E \cap F$ with the intersection topology are related to the completeness of the (LF)-space $E + F$. The present setting provides us with new interesting examples of (LF)-spaces. Recent progress on the study of (LF)-spaces, see [4, 14, 25, 27], is very important in our work. Our main results are Theorems 4 and 7. Examples 5 and 10 show the main difficulties.

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