ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 5, 2006

GENERAL NONDIAGONAL CUBIC HERMITE-PADÉ APPROXIMATION TO THE EXPONENTIAL FUNCTION

CHENG-DE ZHENG AND REN-HONG WANG

ABSTRACT. General nondiagonal cubic Hermite-Padé approximation to the exponential function with coefficient polynomials of degree at most n, m, s, l, respectively is considered. Explicit formulas and differential equations are obtained for the coefficient polynomials. An exact asymptotic expression is obtained for the error function and it is also shown that these generalized Padé-type approximations can be used to asymptotically minimize the expressions on the unit disk.

1. Introduction. We consider approximations of e^{-x} generated by finding polynomials $P_n,\ Q_m,\ R_s$ and S_l so that

$$\begin{aligned}\mathbf{E}_{nmsl}(x) &:= P_n(x)e^{-3x} + Q_m(x)e^{-2x} + R_s(x)e^{-x} + S_l(x)\\ &= O(x^{n+m+s+l+3}),\end{aligned} \tag{1.1}$$

with P_n, Q_m, R_s, S_l being algebraic polynomials of degree at most n, m, s, l, respectively, and P_n has leading coefficient 1. The approximation of e^{-x} is given by

$$\begin{aligned}\delta_{nmsl}(x) := \sqrt[3]{-\frac{b}{2}+\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{a}{3}\right)^3}} + \sqrt[3]{-\frac{b}{2}-\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{a}{3}\right)^3}}\\ -\frac{Q_m}{3P_n},\end{aligned}$$

2000 AMS *Mathematics Subject Classification.* Primary 41A21.
Key words and phrases. Padé-type approximant, cubic Hermite-Padé approximation, asymptotic formula, differential equation.
Supported by the Natl. Natural Sci. Foundation of China (No. 69973010 and 10271022) and the Natural Sci. Foundation of Guangdong Province, China (No. 021755).
Received by the editors on Jan. 28, 2003, and in revised form on July 7, 2003.