ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 5, 2006

SOLUTIONS OF BOUNDARY VALUE PROBLEMS FOR $2n$-ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we consider the higher order differential equation

$$(-1)^n x^{(2n)}(t) = f(t, x(t), x'(t), \dots, x^{(2n-1)}(t)), \quad 0 < t < 1, \tag{0.1}$$

subject to one of the following boundary value conditions

$$x^{(2i)}(1) = 0 \quad \text{for} \quad i = 0, 1, \dots, n-1, \qquad x^{(2i+1)}(0) = 0 \quad \text{for} \quad i = 0, 1, \dots, n-1, \tag{0.2}$$

or

$$x^{(i)}(1) = 0 \quad \text{for} \quad i = 0, 1, \dots, n-1, \qquad x^{(i)}(0) = 0 \quad \text{for} \quad i = n, \dots, 2n-1, \tag{0.3}$$

where $f(t, x_0, x_1, \dots, x_{2n-1})$ is continuous. Sufficient conditions for the existence of at least one solution or positive solution of the BVP (1) and (2) and BVP (1) and (3) are established, respectively. The emphasis in this paper is that f depends on all higher-order derivatives and we allow that the variables $x_0, \dots, x_{2n-1}$ in f have the degrees greater than 1. Examples are given to illustrate the main results.

1. Introduction. Recently, there has been increasing interest in the study of the existence of positive solutions of boundary value problems for second order or higher order ordinary differential equations, we refer the reader to [**5, 7, 9, 12–15, 17–19**] and the monographs [**1–3**].

For the second order case, the existence of positive solutions of boundary value problems for nonlinear differential equations has been

Key words and phrases. Solvability, two-point boundary value problem, higher order differential equation, solution, positive solution.

The first author is supported by NNSF of China (10371103), and the second author is supported by NSF of Educational Committee of Hunan Province of China.

Received by the editors on December 15, 2003, and in revised form on June 9, 2004.