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THE $\{K_i(z)\}_{i=1}^{\infty}$ FUNCTIONS

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ABSTRACT. In this paper we define and study the functions

$$K_i(z) = \frac{{}_1M_0(1;1,z+i-1) - {}_1M_0(1;1,i-1)}{{}_1M_{-1}(1;1,i)}, \quad i \in \mathbf{N},$$

where function ${}_vM_m(s;a,z)$ is defined in [**9**]. We give the recurrence relations, asymptotic and other properties. Also, we give the exponential generating function and representation of $K_i(n)$.

1. Introduction. In 1971, Professor Kurepa, see [**5, 6**], defined the *left factorial* $!n$ as the total number of nodes in a finite tree consisting of n levels with the kth level containing $k!$ nodes, $k = 0, 1, 2, \ldots, n-1$. That is, Kurepa defined $!0 = 0$ and for $n \in \mathbf{N}$

$$!n = \sum_{k=0}^{n-1} k!$$

and extended it to the complex half-plane $\Re(z) > 0$ as

$$!z = \int_0^{+\infty} \frac{t^z - 1}{t - 1}\, e^{-t}\, dt. \tag{1}$$

This function can be extended analytically to the whole complex plane by

$$!z = !(z+1) - \Gamma(z+1), \tag{2}$$

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