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CHARACTERIZABILITY OF PSU$(\mathbf{p+1, q})$ BY ITS ORDER COMPONENT(S)

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ABSTRACT. Order components of a finite group were introduced by Chen [**5**]. It was proved that some finite groups are characterizable by their order components.

In this paper we prove that $PSU(p+1,q)$ is uniquely determined by its order component(s) if and only if $(q+1) \mid (p+1)$. A main consequence of our results is the validity of Thompson's conjecture for the groups $PSU(p+1,q)$ where $(q+1)|(p+1)$.

1. Introduction. Let $\pi(n)$ be the set of prime divisors of n, where n is a positive integer. If G is a finite group, then $\pi(G)$ is defined to be $\pi(|G|)$. By using the orders of elements in G, we construct the prime graph of G as follows.

The *prime graph* $\Gamma(G)$ of a group G is the graph whose vertex set is $\pi(G)$, and two distinct primes p and q are joined by an edge (we write $p \sim q$) if and only if G contains an element of order pq. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and let $\pi_1, \pi_2, \dots, \pi_{t(G)}$ be the connected components of $\Gamma(G)$. If $2 \in \pi(G)$, then we always suppose $2 \in \pi_1$.

Now $|G|$ can be expressed as a product of coprime positive integers m_i, $i = 1, 2, \dots, t(G)$ where $\pi(m_i) = \pi_i$. These integers are called *the order components* of G. The set of order components of G will be denoted by $OC(G)$. Also we call $m_2, \dots, m_{t(G)}$ *the odd order components* of G. The order components of non-abelian simple groups having at least three prime graph components are obtained by Chen [**9**, Tables 1–3]. Similarly the order components of non-abelian simple groups with two order components can be obtained by using the tables

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