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ACCUMULATION POINTS OF THE BOUNDARY OF A CAT(0) SPACE ON WHICH A GROUP ACTS GEOMETRICALLY

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ABSTRACT. In this paper, using a result of Ontaneda, we show that there is no isolated point in the boundary of a CAT(0) space on which a group acts geometrically, i.e., properly and cocompactly by isometries, if the cardinal number of the boundary is greater than two.

1. Introduction and preliminaries. The purpose of this paper is to study boundaries of CAT(0) groups, i.e., the boundary of a CAT(0) space on which a group acts geometrically.

We say that a metric space (X, d) is a *geodesic space* if, for each $x, y \in X$, there exists an isometry $\xi : [0, d(x, y)] \to X$ such that $\xi(0) = x$ and $\xi(d(x, y)) = y$ (such a ξ is called a *geodesic*). Also a metric space (X, d) is said to be *proper* if every closed metric ball is compact.

Let (X, d) be a geodesic space, and let T be a geodesic triangle in X. A *comparison triangle* for T is a geodesic triangle $\overline{T}$ in the Euclidean plane $\mathbf{R}^2$ with same edge lengths as T. Choose two points x and y in T. Let $\bar{x}$ and $\bar{y}$ denote the corresponding points in $\overline{T}$. Then the inequality

$$d(x, y) \le d_{\mathbf{R}^2}(\bar{x}, \bar{y})$$

is called the CAT(0)-*inequality*, where $d_{\mathbf{R}^2}$ is the natural metric on $\mathbf{R}^2$. A geodesic space (X, d) is called a CAT(0) *space* if the CAT(0)-inequality holds for all geodesic triangles T and for all choices of two points x and y in T.

Let (X, d) be a proper CAT(0) space and $x_0 \in X$. The *boundary of X with respect to x_0*, denoted by $\partial_{x_0} X$, is defined as the set of all

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