ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 5, 2006

LIONS-PEETRE'S INTERPOLATION METHODS ASSOCIATED WITH QUASI-POWER FUNCTIONS AND SOME APPLICATIONS

MING FAN

ABSTRACT. This paper concerns some properties of Lions-Peetre's interpolation methods of constants and means associated with quasi-power functions, and their applications in harmonic analysis, martingale inequalities, and geometric properties of Banach spaces. We describe Besov-Orlicz spaces and Triebel-Lizorkin-Orlicz spaces in terms of interpolation and wavelet bases. We study the commutators of quasi-logarithmic operators and singular integral operators, Hankel operators in Schatten-Orlicz classes, martingale inequalities for the φ-variation, and the stability of multi-dimensional uniform rotundity under interpolation.

Many problems in analysis can be formulated in terms of the action of operators on function spaces. Interpolation theory is a very powerful tool for obtaining new estimates from old ones. In [**4**], the author investigated Lions-Peetre's interpolation methods of constants and means associated with quasi-power functions, and established its connection with the real interpolation methods in the sense of Brudnyi-Krugljak. These kinds of interpolation methods are a natural generalization of the classical real interpolation methods and may play an important role in other fields of analysis [**4, 5**]. In the present paper, we will study some properties of the above mentioned interpolation methods, and their applications in harmonic analysis, martingale inequalities and geometric properties of Banach spaces. Some classical results can be carried over in a more general context.

The plan of the paper is as follows. Section 1 includes preliminaries about Brudnyi-Krugljak's and Peetre-Gustavsson's interpolation methods. In Section 2, we formulate some useful results for Lions-Peetre's

2000 AMS *Mathematics Subject Classification.* Primary 46B70, 46M35, Secondary 46E30, 46E35.
Key words and phrases. Besov-Orlicz space, commutator, Lions-Peetre's interpolation method, multi-dimensional uniform rotundity, quasi-power, Schatten-Orlicz class, strong φ-variation, Triebel-Lizorkin-Orlicz space.
Received by the editors on September 30, 2003.