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NONLOCAL BOUNDARY VALUE PROBLEM OF HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS AT RESONANCE

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ABSTRACT. In this paper we consider the following nth order nonlocal boundary value problem at resonance case

$$\begin{aligned} &x^{(n)}(t) = f(t, x(t), x'(t), \dots, x^{(n-1)}(t)), \quad t \in (0,1),\\ &x^{(i)}(0) = 0, \quad i = 0, 1, \dots, n-2,\\ &x^{(n-1)}(1) = \int_0^1 x^{(n-1)}(s)\,dg(s), \end{aligned}$$

where $f : [0,1] \times R^n \to R$ is a continuous function, $g : [0,1] \to [0,\infty)$ is a nondecreasing function with $g(0) = 0$. Under the resonance condition $g(1) = 1$, by applying the coincidence degree theory of Mawhin, we obtain some existence results for the boundary value problems. We also give an example to illustrate our results.

1. Introduction. In this paper, we consider the following nth order nonlocal boundary value problem at resonance case

$$\begin{aligned} &x^{(n)}(t) = f(t, x(t), x'(t), \dots, x^{(n-1)}(t)), \quad t \in (0,1),\\ &x^{(i)}(0) = 0, \quad i = 0, 1, \dots, n-2,\\ &x^{(n-1)}(1) = \int_0^1 x^{(n-1)}(s)\,dg(s), \end{aligned}$$

where $f : [0,1] \times R^n \to R$ is a continuous function, $g : [0,1] \to [0,\infty)$ is a nondecreasing function with $g(0) = 0$. In boundary condition (3), the integral is meant in the Rieman-Stieljes sense.

Similar to [**4, 15**], if the linear equation $x^{(n)}(t) = 0$, with boundary conditions (2), (3) has only zero solution, and the differential operator

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