

ON THE BEHAVIOR OF THE SOLUTIONS FOR CERTAIN FIRST ORDER LINEAR AUTONOMOUS FUNCTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT. Some results are given concerning the behavior of the solutions for scalar first order linear autonomous delay as well as neutral delay differential equations. These results are obtained by the use of two distinct real roots of the corresponding characteristic equation.

1. Introduction. This paper deals with the behavior of the solutions of scalar first order linear autonomous delay differential equations as well as neutral delay differential equations. Our results are obtained via two distinct real roots of the corresponding characteristic equations and are motivated by a result due to Driver [3, see Theorem 2]. The case of delay differential equations is treated in Section 2, while Section 3 is devoted to the case of neutral delay differential equations. Our results for delay differential equations can be derived as a special case from the results for the more general case of neutral delay differential equations, under some additional restrictions. This is the reason for which the case of delay differential equations is considered separately.

Some closely related asymptotic results for delay differential equations or neutral delay differential equations have been given by Driver [3], Driver, Sasser and Slater [6], Graef and Qian [8], Kordonis, Niyianni and Philos [12], Philos [13], and Philos and Purnaras [14, 15], see also Arino and Pituk [1], Driver [4] and Györi [9] for certain related results. We must also refer here to the very recent interesting article by Frasson and Verduyn Lunel [7] concerning the large time behavior of linear functional differential equations.

It is an interesting problem to extend the results of this paper for the more general case of periodic delay differential equations, such as in [13], as well as of periodic neutral delay differential equations, cf. [14].

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