ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 6, 2006

ON ONE-COMPLEMENTED SUBSPACES OF MINKOWSKI SPACES WITH SMOOTH RIESZ NORMS

BAS LEMMENS AND ONNO VAN GAANS

ABSTRACT. In this paper we study one-complemented subspaces of Minkowski spaces. The main objective is to examine norms on $\mathbf{R}^n$ for which every one-complemented subspace has a block basis, i.e., a basis of vectors with mutually disjoint supports. We introduce a collection of norms on $\mathbf{R}^n$ and show that, for these norms, each one-complemented subspace has a block basis. This collection contains, among others, finite sums of ℓ_p-norms, where $1 < p < \infty$ and $p \neq 2$, and their duals. In the proofs an important role is played by the derivative of the (scaled) duality map and, in particular, its behavior near the coordinate planes.

1. Introduction. This paper concerns one-complemented subspaces of Minkowski spaces, that is, subspaces of $\mathbf{R}^n$ that are the range of a projection of norm one. A classic result of Kakutani [**9**] says that if X is a real Banach space of dimension at least three, then X is Euclidean if and only if every subspace of X is one-complemented. This implies that the one-complemented subspaces of a Banach space that is not Euclidean are somehow special. The special nature of these subspaces is manifested in the following result, compare Bohnenblust [**4**] and Lindenstrauss and Tzafriri [**10**, Theorem 2.a.4]. If $\mathbf{R}^n$ is equipped with an l_p-norm, where $1 < p < \infty$ and $p \neq 2$, then a subspace is one-complemented if and only if it is the linear span of a family of vectors with mutually disjoint supports. With this result in mind it is natural to ask for which norms on $\mathbf{R}^n$ the one-complemented subspaces are spanned by vectors with mutually disjoint supports.

This question has been examined in general Banach spaces. It is known, for instance, to have a positive answer for L_p-spaces, where $1 \leq p < \infty$ and $p \neq 2$, see Ando [**1**], Bernau and Lacey [**3**], Douglas [**7**] and Tzafriri [**17**], and for some natural generalizations of L_p-spaces such as Lorenz sequence spaces and Orlicz sequence spaces, see Randrianantoanina [**12–14, 16**] and Jamison, Kamińska and Lewicki [**8**]. On the other hand, there exist one-complemented subspaces of $\mathbf{R}^3$

Accepted by the editors on March 23, 2004.