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NONEXISTENCE OF POSITIVE SOLUTIONS FOR A CLASS OF SEMILINEAR ELLIPTIC SYSTEMS

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ABSTRACT. We consider the system

$$\begin{aligned} -\Delta u &= \lambda\, f(v); \quad x \in \Omega \\ -\Delta v &= \mu\, g(u); \quad x \in \Omega \\ u = 0 &= v; \quad x \in \partial\Omega, \end{aligned}$$

where Ω is a ball in R^N, $N \geq 1$ and $\partial\Omega$ is its boundary, λ, μ are positive parameters bounded away from zero, and f, g are smooth functions that are negative at the origin and grow at least linearly at infinity. We establish the nonexistence of positive solutions when $\lambda\,\mu$ is large. Our proofs depend on energy analysis and comparison methods.

1. Introduction. Consider the system

$$\begin{aligned} -\Delta u &= \lambda\, f(v); \quad x \in \Omega \\ -\Delta v &= \mu\, g(u); \quad x \in \Omega \\ u = 0 &= v; \quad x \in \partial\Omega, \end{aligned} \tag{1.1}$$

where Ω is a smooth bounded region in R^N, $\partial\Omega$ is its boundary, λ, $\mu \geq \varepsilon_0$ where $\varepsilon_0 > 0$, and f and g are smooth functions that grow at least linearly at infinity. Such systems arise naturally as steady states in reaction diffusion processes with unequal diffusion coefficients. It is of great interest to find regions of the parameters involved (diffusion coefficients) for which positive steady states cease to exist. If f(0) and g(0) are positive, then the nonexistence of positive solutions to (1.1) follows rather easily, see Appendix A. However the case when $f(0) < 0$ and $g(0) < 0$ is nontrivial.

The main purpose of this paper is to study this strictly semi-positone case. While the case when Ω is any bounded region remains open,

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