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REMARKS ON SPACES OF REAL RATIONAL FUNCTIONS

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ABSTRACT. Let $\mathrm{RRat}_k(\mathbf{C}P^n)$ denote the space of basepoint-preserving conjugation-equivariant holomorphic maps of degree k from S^2 to $\mathbf{C}P^n$. A map $f : S^2 \to \mathbf{C}P^n$ is said to be full if its image does not lie in any proper projective subspace of $\mathbf{C}P^n$. Let $\mathrm{RF}_k(\mathbf{C}P^n)$ denote the subspace of $\mathrm{RRat}_k(\mathbf{C}P^n)$ consisting of full maps. We first determine $H_*(\mathrm{RRat}_k(\mathbf{C}P^n); \mathbf{Z}/p)$ for all primes p. Then we prove that the inclusion $\mathrm{RF}_k(\mathbf{C}P^n) \hookrightarrow \mathrm{RRat}_k(\mathbf{C}P^n)$ and a natural map $\alpha_{k,n} : \mathrm{RF}_k(\mathbf{C}P^n) \to SO(k)/SO(k-n)$ are homotopy equivalences up to dimensions $k-n$ and $n-1$, respectively.

1. Introduction. Let $\mathrm{Rat}_k(\mathbf{C}P^n)$ denote the space of based holomorphic maps of degree k from the Riemannian sphere $S^2 = \mathbf{C} \cup \infty$ to the complex projective space $\mathbf{C}P^n$. The basepoint condition we assume is that $f(\infty) = [1, \dots, 1]$. Such holomorphic maps are given by rational functions:

(1.1)

$$\mathrm{Rat}_k(\mathbf{C}P^n) = \{(p_0(z), \dots, p_n(z)) : \text{each } p_i(z) \text{ is a monic polynomial over } \mathbf{C} \text{ of degree } k \text{ and such that there are no roots common to all } p_i(z)\}.$$

There is an inclusion $\mathrm{Rat}_k(\mathbf{C}P^n) \hookrightarrow \Omega^2_k \mathbf{C}P^n \simeq \Omega^2 S^{2n+1}$. Segal [**9**] proved that the inclusion is a homotopy equivalence up to dimension $k(2n-1)$. (Throughout this paper, to say that a map $f : X \to Y$ is a homotopy equivalence up to dimension d is intended to mean that f induces isomorphisms in homotopy groups in dimensions less than d, and an epimorphism in dimension d.) Later, the stable homotopy type of $\mathrm{Rat}_k(\mathbf{C}P^n)$ was described in [**3**] as follows. Let

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