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DOMAINS OF DIMENSION 1 WITH INFINITELY MANY SINGULAR MAXIMAL IDEALS

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ABSTRACT. Let Λ be a Noetherian domain of dimension 1 with normalization Γ, and let $\mathfrak{m}$ range through the maximal ideals of Λ. We study the set of possible factorizations of $\Gamma\mathfrak{m}$ into products of maximal ideals of Γ, in the situation where infinitely many such $\mathfrak{m}$ can be "singular," that is, $\Lambda_{\mathfrak{m}} \neq \Gamma_{\mathfrak{m}}$.

1. Introduction. Let Λ be a Noetherian domain of (Krull) dimension 1 with normalization Γ, necessarily a Dedekind domain. Then, for each maximal ideal $\mathfrak{m}$ of Λ, we have a factorization $\Gamma\mathfrak{m} = \prod_{j=1}^{g} \mathfrak{m}_j^{e_j}$ into a product of powers of distinct maximal ideals $\mathfrak{m}_j$ of Γ. The integer e_j is sometimes called the *ramification degree* of $\mathfrak{m}_j$ (over $\mathfrak{m}$ or over Λ). Let f_j denote the *residue degree* of $\mathfrak{m}_j$ (over $\mathfrak{m}$ or over Λ), that is, the dimension of $\Gamma/\Gamma\mathfrak{m}_j$ considered as a vector space over $\Lambda/\mathfrak{m}$. Then we have the *efg*-relation

$$\sum_{i=1}^{g} e_j f_j = \lambda_\Lambda(\Gamma/\Gamma\mathfrak{m}) \tag{1.0.1}$$

where $\lambda_\Lambda(\dots)$ denotes the composition length of the Λ-module $(\dots)$.

In the classical situation—where Γ is module-finite over Λ—the localization $\Lambda_{\mathfrak{m}}$ equals $\Gamma_{\mathfrak{m}}$ for *almost all* $\mathfrak{m}$, i.e., for all but finitely many $\mathfrak{m}$. In other words, the *efg*-relation associated with almost every maximal ideal $\mathfrak{m}$ of Λ is *trivial* ($e_j = f_j = g = 1$).

Suppose now that Γ is not module-finite over Λ. Then infinitely many maximal ideals of Λ can have nontrivial *efg*-relations. An instance of this was given by Hochster [**2**]. Our main result is that there is no restriction whatsoever on the *efg*-relations that can occur in this non-finite situation (Theorem 3.2). Note that each individual *efg*-relation

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