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BASIC FUNCTIONAL EQUATIONS OF THE ROGERS-RAMANUJAN FUNCTIONS

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ABSTRACT. The Rogers-Ramanujan functions satisfy some basic functional equations. We prove and use them to produce some identities that Ramanujan recorded.

1. Introduction. The Rogers-Ramanujan functions in the title are defined by

$$G(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n},$$
$$H(q) := \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q;q)_n},$$

where

$$(a;q)_n := \prod_{k=0}^{n-1} (1 - aq^k), \quad |q| < 1. \tag{1.1}$$

These functions satisfy the Rogers-Ramanujan identities [**3**], [**4**, pp. 214–215], [**6**],

$$G(q) = \frac{1}{(q;q^5)_\infty (q^4;q^5)_\infty}, \tag{1.2}$$
$$H(q) = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty}, \tag{1.3}$$

where

$$(a;q)_\infty := \lim_{n\to\infty} (a;q)_n. \tag{1.4}$$

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