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A GENERALIZED HAWKINS SIEVE AND PRIME K-TUPLETS

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ABSTRACT. The Hawkins random sieve, obtained from a simple probabilistic variation of Eratoshenes's sieve, provides a compelling model for the primes. Building on the Hawkins' sieve, we introduce a general random sieve, and prove analogs of both the Prime Number Theorem and Mertens' theorem. Applications include a new probabilistic model for prime k-tuplets.

1. Introduction.

1.1 Purpose. In this paper, we introduce a natural generalization of the Hawkins' random sieve, and prove analogs of both the Prime Number Theorem (PNT) and Mertens' theorem in the more general setting. As an application we present a new probabilistic model for prime k-tuplets.

1.2 Background. When faced with the complexity of prime distribution theory, it is tempting to employ mathematical models. One of the most compelling models for the prime numbers is known as the *Hawkins' primes*. The Hawkins' model, first introduced by David Hawkins [**13**], is based on a simple stochastic variation of the sieve of Eratosthenes. Hawkins' sieve works as follows: Starting with all natural numbers two and larger, we identify $X_1 = 2$ as our first 'sieving number.' In the first step we independently sieve numbers from our list with probability $1/X_1$, and identify X_2 as the smallest surviving number which is larger than X_1. In the second step, we sieve numbers from our remaining list with probability $1/X_2$ and identify X_3 as the smallest surviving number which is larger than X_2. If we carry on with the process, we produce a list $\{X_1, X_2, \dots, \}$ of sieving numbers which are called Hawkins' primes.

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