ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 2, 2007

EQUAL SUMS OF SIXTH POWERS AND QUADRATIC LINE COMPLEXES

MASATO KUWATA

1. Introduction. This paper is concerned with the Diophantine equation:

$$x^6 + y^6 + z^6 = u^6 + v^6 + w^6. \tag{1.1}$$

We present a relation between this equation and Kummer's quartic surfaces through the theory of quadratic line complexes.

To this date there have been many numerical solutions to (1.1) discovered by various computer searches. (See Section 7 for more historical details.) A large part of them also satisfy the quadratic equation

$$x^2 + y^2 + z^2 = u^2 + v^2 + w^2. \tag{1.2}$$

Furthermore, Bremner [**1**] shows that among those simultaneous solutions, most also satisfy the system of equations:

$$\begin{cases} x^2 + xu - u^2 = w^2 + wz - z^2, \\ y^2 + yv - v^2 = u^2 + ux - x^2, \\ z^2 + zw - w^2 = v^2 + vy - y^2. \end{cases} \tag{1.3}$$

Note that we recover (1.1) by cubing each equation in (1.3) and adding them.

Geometrically, this can be seen as follows. Let V_4 be the fourfold defined by (1.1). Many of the rational points of V_4 are contained in the subthreefold V_3 cut out by the quadric (1.2). The system of equations (1.3) determines a $K3$ surface K_B contained in V_3, and most of the known rational points of V_3 are contained in K_B. Bremner [**1**] investigated this $K3$ surface geometrically in depth. Among others, he gave a theoretical method to find all smooth parametric solutions.

AMS *Mathematics Subject Classification.* Primary 14J28, 11D41.
Received by the editors on November 8, 2004.