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# ON THE IRREDUCIBILITY OF A TRUNCATED BINOMIAL EXPANSION

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**1. Introduction.** For positive integers $k$ and $n$ with $k \leq n-1$, define

$$P_{n,k}(x) = \sum_{j=0}^{k} \binom{n}{j} x^j.$$

In the case that $k = n-1$, the polynomial $P_{n,k}(x)$ takes the form

$$P_{n,n-1}(x) = (x+1)^n - x^n.$$

If $n$ is not a prime, $P_{n,n-1}(x)$ is reducible over $\mathbf{Q}$. If $n = p$ is prime, the polynomial $P_{n,n-1}(x) = P_{p,p-1}(x)$ is irreducible as Eisenstein's criterion applies to the reciprocal polynomial $x^{p-1}P_{p,p-1}(1/x)$. This note concerns the irreducibility of $P_{n,k}(x)$ in the case where $1 \leq k \leq n-2$. Computations for $n \leq 100$ suggest that in this case $P_{n,k}(x)$ is always irreducible. We will not be able to establish this but instead give some results which give further evidence that these polynomials are irreducible.

The problem arose during the 2004 MSRI program on *Topological aspects of real algebraic geometry*, in the context of work by Inna Scherbak in investigations of the Schubert calculus in Grassmannians. She had observed that the roots of any given $P_{n,k}(x)$ are simple. This follows from the identity

$$P_{n,k}(x) - (x+1)\,\frac{P'_{n,k}(x)}{n} = \binom{n-1}{k} x^k.$$

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