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# NONOSCILLATORY CRITERIA FOR SECOND-ORDER NONLINEAR DIFFERENCE EQUATIONS

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ABSTRACT. In this paper, we obtain some nonoscillatory theories of the second-order nonlinear difference equation

$$\triangle(r_n(\triangle x_n)^\alpha) + f(n+1,\ x_{n+1}) = 0, \quad n \in \mathbf{N}$$

where $\alpha$ is a quotient of positive odd integers, $r_n > 0$ for $n \in \mathbf{N}$ and $f \in C(\mathbf{N} \times \mathbf{R},\ \mathbf{R})$.

**1. Introduction.** Consider the following second-order difference equation

$$\triangle(r_n(\triangle x_n)^\alpha) + f(n+1,\ x_{n+1}) = 0, \quad n \in \mathbf{N} \tag{1}$$

where $\alpha$ is a quotient of positive odd integers, $\triangle x_n = x_{n+1} - x_n$, $r_n > 0$ for $n \in \mathbf{N}$ and $f \in C(\mathbf{N} \times \mathbf{R},\ \mathbf{R})$.

A solution of (1) is called nonoscillatory if it is either eventually positive or eventually negative; otherwise, it is called oscillatory.

In [**6**–**10**], many good results for nonoscillatory solutions of differential equations corresponding to (1) were obtained, but in the results the condition where $f(t,\ x)$ is either linear or quasi-linear was adopted. So far, very few results for nonoscillation of (1) with generally nonlinear term have been obtained. In this paper, by using the methods in the proof of [**1**], we discuss nonoscillatory solutions of (1) and obtain the following results.

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