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CONTINUITY OF HOMOMORPHISMS AND DERIVATIONS ON NORMED ALGEBRAS WHICH ARE TENSOR PRODUCTS OF ALGEBRAS WITH INVOLUTION

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ABSTRACT. We prove that, if A is a normed $*$-algebra of the form $B \otimes C$ for some central simple finite-dimensional algebra B with involution different from $\pm I_B$ and some algebra C with involution and a unit, then homomorphisms from A to normed algebras and derivations from A to normed A-bimodules are continuous whenever they are continuous on the hermitian part of A. When A is associative, some additional information is given.

1. Introduction. The aim of this paper is to study the automatic continuity of some homomorphisms and derivations with "arbitrary range" and whose domains are normed $*$-algebras over $\mathbf{K}$ ($= \mathbf{R}$ or $\mathbf{C}$) of the type $B \otimes C$. Here $B \otimes C$ stands for the algebraic tensor product of algebras B and C, each of them endowed with a (linear) involution. Our achievements in this line are collected in two independent results of the same flavor, namely Theorems 3 and 5, and are derived from Theorem 2, which is the main result in this paper. In the last quoted theorem we show that, if A is a normed $*$-algebra of the form $B \otimes C$ for some central simple finite-dimensional algebra B with involution different from $\pm I_B$, and some algebra C with involution and a unit, then two algebra norms on A making the tensor involution continuous are equivalent whenever they are equivalent on the hermitian part of A. As a consequence, if $n \geq 2$, if C is an algebra over $\mathbf{K}$ with involution and a unit, if $M_n(C)$ denotes the algebra of all $n \times n$ matrices with entries in C, and if we endow $M_n(C)$ with the standard involution (consisting in transposing a given matrix and applying the involution of C to each entry), then two algebra norms on $M_n(C)$ making its involution continuous are equivalent whenever they are equivalent on the hermitian part of $M_n(C)$. The fact just reviewed can be reformulated as follows. If A is a normed $*$-algebra over $\mathbf{K}$ which, algebraically regarded, is of

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