ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 3, Fall 2002

SPHERE-FOLIATED MINIMAL AND CONSTANT MEAN CURVATURE HYPERSURFACES IN SPACE FORMS AND LORENTZ-MINKOWSKI SPACE

SUNG-HO PARK

ABSTRACT. We prove that a sphere-foliated minimal or constant mean curvature hypersurface in hyperbolic space of dimension ≥ 5 is one of the following: hypersurface of rotation around a geodesic, geodesic hyperplane, horosphere, equidistant hypersurface, or a geodesic sphere in the upper half-space model. And we show that a sphere-foliated minimal or constant mean curvature hypersurface in sphere of dimension ≥ 5 is either a hypersurface of rotation or a hypersphere.

We also show that a hypersurface of nonzero constant mean curvature in Lorentz-Minkowski space foliated by spheres in space-like hyperplanes is either a hypersurface of rotation or a pseudo-hyperbolic space and that maximal space-like hypersurfaces foliated by spheres in hyperplanes are rotational if the ambient space has dimension ≥ 4.

1. Introduction. A hypersurface M of $\mathbf{R}^{n+1}$ is said to be *sphere-foliated* if there is a one-parameter family of hyperplanes that meet M in round $(n-1)$-spheres. A circle-foliated surface is called *cyclic*.

Examples of cyclic constant mean curvature (CMC) surfaces are the Delaunay's surfaces and the spheres. While Delaunay's surfaces are rotational, spheres admit plenty of nonrotational foliations by circles. Nitsche claimed that *all cyclic surfaces of nonvanishing constant mean curvature are surfaces of rotation* [**12**]. Though his claim is right, his proof is incomplete. Our first aim in Section 2 is to give a complete proof of the following modified form of Nitsche's claim.

Theorem 1. *If M is a cyclic surface of nonzero constant mean curvature, then it is either a surface of rotation or a sphere.*

As a consequence of this theorem, we see that there is no cyclic surface of nonzero constant mean curvature spanning two *non-coaxial* circles

Author supported in part by BK21.
Received by the editors on April 25, 2000, and in revised form on October 24, 2000.