ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 3, Fall 2002

ON LINEAR MEANS OF MULTIPLE FOURIER INTEGRALS DEFINED BY SPECIAL DOMAINS

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ABSTRACT. Weak and strong estimates in weighted L^p spaces are obtained for linear means of Fourier integrals defined by a single function with support in a specially organized set.

Introduction. For a function f integrable on the n-dimensional Euclidean space $\mathbf{R}^n$, written $f \in L^1(\mathbf{R}^n)$, its Fourier transform is well defined

$$\hat{f}(x) = \int_{\mathbf{R}^n} f(u)e^{-ixu}\,du,$$

where $x = (x_1, x_2, \dots, x_n)$, $u = (u_1, u_2, \dots, u_n) \in \mathbf{R}^n$ and $xu = x_1u_1 + x_2u_2 + \cdots + x_nu_n$. Let

$$\int_D \hat{f}(x)e^{iux}\,dx$$

be the partial Fourier integral defined by a set D. The behavior of partial Fourier integrals with respect to a specifically organized family of such sets characterizes approximation properties of f. It is natural to define such a family as a sequence of dilations of a fixed set D. This has been extensively studied when D is the cube (cubic case)

$$D = \{x \in \mathbf{R}^n : |x_j| \le 1,\ j = 1, 2, \dots, n\},$$

or the ball (spherical case)

$$D = \{x \in \mathbf{R}^n : |x| = (x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2} \le 1\}.$$

Their R-dilations are

$$RD = \{x \in \mathbf{R}^n : |x_j| \le R,\ j = 1, 2, \dots, n\}$$

2000 *Mathematics Subject Classification.* 42B25, 42B35, 42B10.
Received by the editors on June 4, 2000, and in revised form on September 27, 2000.