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A NEW A_n EXTENSION OF RAMANUJAN'S ${}_1\psi_1$ SUMMATION WITH APPLICATIONS TO MULTILATERAL A_n SERIES

STEPHEN C. MILNE AND MICHAEL SCHLOSSER

ABSTRACT. In this article we derive some identities for multilateral basic hypergeometric series associated to the root system A_n. First, we apply Ismail's [**15**] argument to an A_n q-binomial theorem of Milne [**25**, Theorem 5.42] and derive a new A_n generalization of Ramanujan's ${}_1\psi_1$ summation theorem. From this new A_n ${}_1\psi_1$ summation and from an A_n ${}_1\psi_1$ summation of Gustafson [**9**], we deduce two lemmas for deriving simple A_n generalizations of bilateral basic hypergeometric series identities. These lemmas are closely related to the Macdonald identities for A_n. As samples for possible applications of these lemmas, we provide several A_n extensions of Bailey's ${}_2\psi_2$ transformations, and several A_n extensions of a particular ${}_2\psi_2$ summation.

1. Introduction. The theory of basic hypergeometric series (cf. [**8**]), consists of many known summation and transformation formulas. The most important of these is probably the q-binomial theorem, a summation first discovered by Cauchy [**6**]. Surprisingly, the q-binomial theorem admits a bilateral generalization, the ${}_1\psi_1$ summation theorem, first discovered by Ramanujan [**11**]. Other important identities for basic hypergeometric series include the q-Gauß summation and Heine's ${}_2\phi_1$ transformations. These and many other basic hypergeometric series identities conspicuously appear in combinatorics and in related areas, such as number theory, statistics, physics and representation theory of Lie algebras, see Andrews [**1**].

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