ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 2, Summer 2002

DIVISION PROBLEM OF MOMENT FUNCTIONALS

J.H. LEE AND K.H. KWON

ABSTRACT. For a quasi-definite moment functional σ and nonzero polynomials $A(x)$ and $D(x)$, we define another moment functional τ by the relation

$$D(x)\tau = A(x)\sigma.$$

In other words, τ is obtained from σ by a linear spectral transform. We find necessary and sufficient conditions for τ to be quasi-definite when $D(x)$ and $A(x)$ have no nontrivial common factor. When τ is also quasi-definite, we also find a simple representation of orthogonal polynomials relative to τ in terms of orthogonal polynomials relative to σ. We also give two illustrative examples when σ is the Laguerre or Jacobi moment functional.

1. Introduction. Let σ be a quasi-definite moment functional, i.e., a linear function on **P**, the space of polynomials in one variable, satisfying the Hamburger condition: $\Delta_n := |[\sigma_{i+j}]_{i,j=0}^n| \neq 0$, $n \geq 0$, where $\sigma_n := \langle \sigma, x^n \rangle$, $n \geq 0$, are the moments of σ. Then the monic orthogonal polynomial system (MOPS) $\{P_n(x)\}_{n=0}^\infty$, relative to σ, is given by

(1.1)

$$P_0(x) = 1 \quad \text{and } P_n(x) = \frac{1}{\Delta_{n-1}} \begin{vmatrix} \sigma_0 & \sigma_1 & \cdots & \sigma_n \\ \sigma_1 & \sigma_2 & \cdots & \sigma_{n+1} \\ \vdots & \vdots & & \vdots \\ \sigma_{n-1} & \sigma_n & \cdots & \sigma_{2n-1} \\ 1 & x & \cdots & x^n \end{vmatrix}, \quad n \geq 1.$$

However, in the computational viewpoint, the formula (1.1) is of little practical value for large n. Instead we might use the three-term recurrence relation satisfied by any MOPS

$$P_{n+1}(x) = (x-b_n)P_n(x) - c_n P_{n-1}(x), \quad n \geq 0, (P_{-1}(x) = 0, P_0(x) = 1)$$

1991 AMS *Mathematics subject classification.* 33C45.
Keywords and phrases. Quasi-definiteness, moment functionals, division problem.
This work is partially supported by BK-21 project and KOSEF (99-2-101-001-5).
Received by the editors on October 26, 2000, and in revised form on July 3, 2001.