

## TRANSMUTATION KERNELS FOR THE LITTLE $q$ -JACOBI FUNCTION TRANSFORM

ERIK KOELINK AND HJALMAR ROSENGREN

**ABSTRACT.** The little  $q$ -Jacobi function transform depends on three parameters. An explicit expression as a sum of two very well-poised  ${}_8W_7$ -series is derived for the dual transmutation kernel relating little  $q$ -Jacobi function transforms for different parameter sets. A product formula for the dual transmutation kernel is obtained. For the inverse transform, the transmutation kernel is given as a  ${}_3\varphi_2$ -series, and a product formula as a finite sum is derived. The transmutation kernel gives rise to intertwining operators for the second order hypergeometric  $q$ -difference operator, which generalize the intertwining operators arising from a Darboux factorization.

**1. Introduction.** The Jacobi transform is an integral transform on the positive half-line with a hypergeometric  ${}_2F_1$ -series as its kernel. This transform is a two-parameter extension of the Fourier-cosine transform and the Mehler-Fock transform and also contains the Hankel transform as a limit case. The inversion formula for the Jacobi transform can be found explicitly in several ways, using asymptotics, spectral analysis, group theory or intertwining properties. This transform has a long history and we refer the reader to the survey paper [13] by Koornwinder.

There are several levels of  $q$ -analogues of the Jacobi function and of the corresponding transform pair (see [9] for an overview and references). Here we consider the so-called little  $q$ -Jacobi function and the corresponding transform. The little  $q$ -Jacobi function transform has been studied by Takeuchi, Masuda and Ueno [7, 6] as the (spherical) Fourier transform on the quantum  $SU(1,1)$  group using the interpretation of the little  $q$ -Jacobi functions on the quantum  $SU(1,1)$  group using the interpretation of the little  $q$ -Jacobi functions as matrix elements of unitary irreducible representations of  $U_q(\mathfrak{su}(1,1))$  (see [14]). On the other hand, the little  $q$ -Jacobi function transform occurs when

---

*Keywords and phrases.* Summation formula, transmutation kernels, product formula, little  $q$ -Jacobi function, intertwiner, fractional  $q$ -integral operator.  
2000 AMS *Mathematics subject classification.* 33D15, 33D45, 47B36.  
Received by the editors on October 31, 2000.