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AN INVERSE TO THE ASKEY-WILSON OPERATOR

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ABSTRACT. We study properties of the kernel of a right inverse of the Askey-Wilson divided difference operator on L^2 weighted with the weight function of the continuous q-Jacobi polynomials. This operator is embedded in a one-parameter family of integral operators, denoted by $\mathcal{D}_q^{-t}$ whose kernel is related to the Poisson kernel. It is shown that as $t \to 1^-$, the t-commutator $(\mathcal{D}_q\mathcal{D}_q^{-t} - t\mathcal{D}_q^{-t}\mathcal{D}_q)f$ tends to the constant term in the orthogonal expansion of f in continuous q-Jacobi polynomials.

1. Introduction. Given a function $f(x)$ with $x = \cos\theta$, then $f(x)$ can be viewed as a function of $e^{i\theta}$. Let

$$\breve{f}(e^{i\theta}) := f(x), \quad x = \cos\theta. \tag{1.1}$$

In this notation the Askey-Wilson divided difference operator $\mathcal{D}_q$ [**4**] is defined by

$$(\mathcal{D}_q f)(x) := \frac{\breve{f}(q^{1/2}e^{i\theta}) - \breve{f}(q^{-1/2}e^{i\theta})}{\breve{e}(q^{1/2}e^{i\theta}) - \breve{e}(q^{-1/2}e^{i\theta})}, \tag{1.2}$$

where $e(x) = x$. It follows easily from (1.2) that

$$(\mathcal{D}_q f)(x) = \frac{\breve{f}(q^{1/2}e^{i\theta}) - \breve{f}(q^{-1/2}e^{i\theta})}{i(q^{1/2} - q^{1/2})\sin\theta}. \tag{1.3}$$

The operator $\mathcal{D}_q$ was introduced in [**4**] and is a q-analogue of the differentiation operator d/dx. Note that $\mathcal{D}_q$ remains invariant if q is

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