INVERSION TECHNIQUES AND
COMBINATORIAL IDENTITIES:
BALANCED HYPERGEOMETRIC SERIES

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Dedicated to my teacher L.C. Hsu on the occasion of his 80th birthday

ABSTRACT. Following the earlier works on Inversion techniques and combinatorial identities, the duplicate form of the Gould-Hsu [18] inversion theorem is constructed. As applications, several terminating balanced hypergeometric formulas are demonstrated, including those due to Andrews [3], which have been the primary stimulation to the present research. Encouraged by the recent work of Standon [23], we establish two higher hypergeometric evaluations with three additional parameters, which specialize further to over two hundred hypergeometric identities.

For a complex \( c \) and a natural number \( n \), denote the rising shifted-factorial by

\[
(0.1a) \quad (c)_0 = 1, \quad (c)_n = c(c + 1) \cdots (c + n - 1), \quad n = 1, 2, \ldots.
\]

Following Bailey [8], the hypergeometric series, for an indeterminate \( z \) and two nonnegative integers \( m \) and \( n \), is defined by

\[
(0.1b) \quad \binom{a_0, a_1, \ldots, a_n}{b_1, \ldots, b_m}_z = \sum_{k=0}^{\infty} \frac{(a_0)_k(a_1)_k \cdots (a_n)_k}{k!(b_1)_k \cdots (b_m)_k} z^k,
\]

where \( \{a_i\} \) and \( \{b_j\} \) are complex parameters such that no zero factors appear in the denominators of the summands on the right-hand side. When the variable \( z = 1 \), it will be omitted from the hypergeometric notation. If one of the numerator parameters \( \{a_k\} \) is a negative integer, then the series becomes terminating, which reduces to a polynomial in \( z \).


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