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# INVERSION TECHNIQUES AND COMBINATORIAL IDENTITIES: BALANCED HYPERGEOMETRIC SERIES

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*Dedicated to my teacher L.C. Hsu on the occasion of his 80th birthday*

ABSTRACT. Following the earlier works on *Inversion techniques and combinatorial identities*, the duplicate form of the Gould-Hsu [**18**] inversion theorem is constructed. As applications, several terminating balanced hypergeometric formulas are demonstrated, including those due to Andrews [**3**], which have been the primary stimulation to the present research. Encouraged by the recent work of Standon [**23**], we establish two higher hypergeometric evaluations with three additional parameters, which specialize further to over two hundred hypergeometric identities.

For a complex $c$ and a natural number $n$, denote the rising shifted-factorial by

$$(c)_0 = 1, \quad (c)_n = c(c+1)\cdots(c+n-1), \quad n = 1, 2, \dots . \tag{0.1a}$$

Following Bailey [**8**], the hypergeometric series, for an indeterminate $z$ and two nonnegative integers $m$ and $n$, is defined by

$$ {}_{1+n}F_m \left[ \begin{matrix} a_0, & a_1, & \cdots, & a_n \\ & b_1, & \cdots, & b_m \end{matrix} ; z \right] = \sum_{k=0}^{\infty} \frac{(a_0)_k (a_1)_k \cdots (a_n)_k}{k! (b_1)_k \cdots (b_m)_k} z^k, \tag{0.1b}$$

where $\{a_i\}$ and $\{b_j\}$ are complex parameters such that no zero factors appear in the denominators of the summands on the righthand side. When the variable $z = 1$, it will be omitted from the hypergeometric notation. If one of the numerator parameters $\{a_k\}$ is a negative integer, then the series becomes terminating, which reduces to a polynomial in $z$.

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