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ON SAMPLING ASSOCIATED WITH SINGULAR STURM-LIOUVILLE EIGENVALUE PROBLEMS: THE LIMIT-CIRCLE CASE

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ABSTRACT. Sampling expansions are derived for solutions of second order singular Sturm-Liouville eigenvalue problems in the limit-circle case. In this setting special functions with continuous parameter satisfying such problems will be sampled in terms of these functions with a discrete parameter, sometimes orthogonal polynomials. As examples, the Legendre and Bessel functions are sampled. Sampling expansions of the associated integral transforms are also given. The analysis makes use of the approach derived by Fulton [**13**] and extends the range of examples studied by Butzer-Schöttler, Everitt and Zayed. A fully new example is the appearance of the Legendre function of the second kind in the analysis of sampling theory.

1. Introduction. Let $P_w(z)$, $w, z \in \mathbf{C}$, with $|z| < 1$ denote the Legendre function of the first kind, i.e. (see [**20, 23**])

$$P_w(z) = {}_2F_1\bigg(-w, w+1; 1; \frac{1-z}{2}\bigg), \tag{1.1}$$

where ${}_2F_1(a, b; c; \cdot)$ is the Gauss hypergeometric series. The following sampling expansion for $P_w(z)$ was derived in [**7**], (see also [**4**–**6**]):

$$\begin{aligned}\frac{P_{w-1/2}(x)}{w^2-1/4} &= \sum_{k=1}^{\infty} \frac{P_k(x)}{k(k+1)} \frac{(2k+1)\sin\pi(w-(k+(1/2)))}{\pi(w^2-(k+(1/2))^2)} \\ &\quad + \frac{\sin\pi(w+(1/2))}{\pi(w^2-(1/4))}\bigg\{\frac{1}{w^2-(1/4)} - \log\bigg(\frac{2}{1+x}\bigg)+1\bigg\},\end{aligned} \tag{1.2}$$

where $w \in \mathbf{R} - \{1/2\}$, $x \in (-1, 1]$ and $P_k(\cdot)$ are the Legendre polynomials. The convergence of (1.2) is in the L^2-norm with respect to

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