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SOME q-ORTHOGONAL POLYNOMIALS AND RELATED HANKEL DETERMINANTS

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1. Introduction. This paper grew out of some experiments using the computer algebra MAPLE. Let the function $f(t)$ have the Taylor series development

$$f(t) = \sum_{n=0}^{\infty} f_n t^n,$$

which we assume converges in a neighborhood of the origin. The coefficients f_n may be interpreted as the moments of a suitable function, actually the complex moments

$$f_n = L(z^n) = \frac{1}{2\pi i} \oint z^n \left(\frac{f(1/z)}{z} \right) dz,$$

where the path of integration is, say, a circle centered at the origin with a suitably large radius. Using the construction given in [**1**] and the Gram determinants

$$G_N = \begin{vmatrix} f_0 & f_1 & \cdots & f_N \\ f_1 & f_2 & \cdots & f_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ f_N & f_{N+1} & \cdots & f_{2N} \end{vmatrix},$$

one may construct the monic polynomials, call them $P_N(x)$, that are orthogonal to the distribution which gives these moments.

$$P_N(x) = \frac{1}{G_{N-1}} \begin{vmatrix} f_0 & f_1 & \cdots & f_N \\ f_1 & f_2 & \cdots & f_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ f_{N-1} & f_N & \cdots & f_{2N-1} \\ 1 & x & \cdots & x^N \end{vmatrix}.$$

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