ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 1, Spring 2002

THE HAUSDORFF DIMENSION OF THE NONDIFFERENTIABILITY SET OF A NONSYMMETRIC CANTOR FUNCTION

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ABSTRACT. Each choice of numbers a and c in the segment $(0,(1/2))$ produces a Cantor set C_{ac} by recursively removing segments from the interior of the interval $[0,1]$ so that intervals of relative length a and c remain on the left and right sides of the removed segment, respectively. A Cantor function Φ_{ac} is obtained from C_{ac} in much the same way that the standard Cantor function, Φ, is obtained from the Cantor ternary set. When $a = c = (1/3)$, C_{ac} is the Cantor ternary set, C, and Φ_{ac} is the standard Cantor function, Φ. The derivative of Φ is zero off C, and the upper derivative is infinite on C; the set $N = \{x \in C \mid$ the lower derivative of Φ is finite$\}$ has Hausdorff dimension $[\ln 2/\ln 3]^2$. In this paper similar results are established for N_{ac}, the nondifferentiability set of Φ_{ac}. The Hausdorff dimension of N_{ac} is the maximum of the real numbers satisfying the following equations: $x(\ln(1/c))^2 = \ln((a+c)/c)\ln((a/c)^x+1)$, and $x(\ln(1/a))^2 = \ln((a+c)/a)\ln((c/a)^x+1)$.

1. Introduction. For any numbers a and c satisfying $0 < a$, $c < 1$, we generate a Cantor set in $[0,1]$ by recursively removing open intervals of relative length $b = 1-a-c$ so that closed intervals of relative length a and c remain to the left and right, respectively, of the removed interval:

$$C_{ac}^0 = [0,1],$$
$$C_{ac}^1 = [0,a] \cup [1-c,1],$$
$$C_{ac}^2 = [0,a^2] \cup [a-ac,a] \cup [1-c,1-c+ac] \cup [1-c^2,1],$$

etc., and $C_{ac} = \cap_{n\geq 1} C_{ac}^n$. We will refer to the set C_{ac}^n as the nth *stage in the construction of* C_{ac} and the 2^n closed intervals comprising C_{ac}^n will be called *stage* n *black intervals* or nth *stage black intervals*. The closures of the open intervals removed at various stages in the construction of C_{ac} will be called *complementary intervals* of the appropriate stage.

Received by the editors on December 13, 2000.