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NUMERICAL OPERATIONAL CALCULUS FOR MATRICES WITH APPLICATIONS TO MECHANICAL AND MATHEMATICAL PROBLEMS

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ABSTRACT. The calculation of the matrix exponential e^A is important in many problems of mechanics and applied mathematics. In this paper its calculation is based on the Dunford-Taylor integral representation. As a contour line, a polygonal is chosen where the eigenvalues of A lie in its interior, the contour integral is evaluated numerically by a summed Gaussian quadrature formula, and estimates of the discretization error for a mechanical problem are given which are optimal in a certain sense, and which prove the convergence of the described method. It is shown theoretically that the method – called Numerical Operational Calculus – is superior to the methods known so far for sparse matrices of large order, a situation which often occurs in applications. The theoretical considerations are confirmed by numerical tests for the free-vibration problem of a multi-mass vibration chain. We stress that the damping matrix need not be proportional to the mass and/or stiffness matrix. Also, the method is applied to a series of problems from mathematics showing its wide range of applicability.

0. Introduction. The calculation of matrix functions by contour integrals has been widely used in recent years for problems from physics (cf., e.g., [**1**], [**5**] and [**13**]).

In this paper, we want to carry over this method to the computation of the fundamental matrix, which has not yet been done, as far as we know. As opposed to [**1**], [**5**] and [**13**], we give estimates for the discretization error.

The paper contains two chapters, namely Chapter I: Theory and Chapter II: Applications. Chapter I consists of Sections 1 and 2, and Chapter II of Sections 3 and 4.

In Section 1, we start with Cauchy's integral theorem and the numerical evaluation of the integral over an interval, followed by a summed quadrature formula and the evaluation of the contour integral over a closed polygonal. These results serve as a preparation to the next section.