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ENUMERATIVE TRIANGLE GEOMETRY PART 1: THE PRIMARY SYSTEM, S

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1. Introduction. This article presents a procedure for counting Euclidean constructible objects in triangle geometry–points and lines in Part 1, together with circles in Part 2. If A, B, C are the vertex angles of the reference triangle, then each object is a function $F(A, B, C)$, both as a construction and in the form of homogeneous coordinates. The counting procedure depends on the fact that objects occur in sets formally of size 6:

$$F(A,B,C),\ F(B,C,A),\ F(C,A,B),$$
$$F(A,C,B),\ F(B,A,C),\ F(C,B,A).$$

For homogeneous coordinates, we shall use trilinears. Basic lore on trilinears, presented in the references, is assumed.

2. Primary system, S. Let $S_0 := \{A, B, C\}$, where the object $A := 1 : 0 : 0$ may be interpreted as the point with trilinears $1 : 0 : 0$ or as the line with equation $u\alpha + v\beta + w\gamma = 0$ having coefficients $u : v : w = 1 : 0 : 0$, i.e., the line BC, which may be called the A-sideline just as the point having trilinears $1 : 0 : 0$ is the A-vertex.

Three operations, or *opera* (singular *opus*) will now be defined and eventually applied to objects in S_0 and in succeeding generations of objects. Let $U = u : v : w$ and $X = x : y : z$.

$$\text{(1)}\quad \text{Opus 1}: \quad U \cdot X := \begin{cases} U & \text{if } X = U \\ vz - wy : wx - uz : uy - vx & \text{if } X \neq U; \end{cases}$$

if U and X are interpreted as distinct points, then $U \cdot X$ is their line, and if U and X are interpreted as distinct lines, then $U \cdot X$ is their point of intersection.

$$\text{(2)}\quad \text{Opus 2}: \quad U \| X := v(ay - bx) + w(az - cx) : w(bz - cy) + u(bx - ay) : u(cx - az) + v(cy - bz)$$

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