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SOME APPROXIMATION THEOREMS VIA STATISTICAL CONVERGENCE

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ABSTRACT. In this paper we prove some Korovkin and Weierstrass type approximation theorems via statistical convergence. We are also concerned with the order of statistical convergence of a sequence of positive linear operators.

1. Introduction and Background. Let E be a subset of $\mathbf{N}$, the set of all natural numbers. The density of E is defined by $\delta(E) := \lim_n (1/n) \sum_{j=1}^n \chi_E(j)$ whenever the limit exists, where χ_E is the characteristic function of E. The number sequence $\alpha = (\alpha_k)$ is statistically convergent to the number L if, for every $\varepsilon > 0$, $\delta\{k \in \mathbf{N} : |\alpha_k - L| \geq \varepsilon\} = 0$, [**4**], or equivalently there exists a subset $K \subseteq \mathbf{N}$ with $\delta(K) = 1$ and $n_0(\varepsilon)$ such that $k > n_0$ and $k \in K$ imply that $|\alpha_k - L| < \varepsilon$, [**5**], [**10**], [**12**]. In this case we write $st - \lim \alpha_k = L$. It is known that any convergent sequence is statistically convergent, but not conversely. For example, the sequence (α_n) defined by $\alpha_n = \sqrt{n}$ if n is square and $\alpha_n = 0$ otherwise has the property that $st - \lim \alpha_n = 0$.

Some basic properties of statistical convergence are exhibited in [**2**], [**12**], [**13**]. Over the years this concept has been examined in number theory [**3**], trigonometric series [**14**], probability theory [**7**], optimization [**11**], measure theory [**10**] and summability theory [**2**], [**5**], [**6**].

Statistical convergence has not been examined in approximation theory so far. It is the purpose of this paper to consider it in some problems of approximation theory. Especially we shall be concerned with the Korovkin approximation theory which deals with the problem of approximation of function f by the sequence $(A_n(f, x))$ where (A_n) is a sequence of positive linear operators [**8**], [**9**].

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