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THE EXTENSION OF p-ADIC COMPACT OPERATORS

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ABSTRACT. This paper is devoted to study the non-Archimedean locally convex spaces X having the following property: For all non-Archimedean locally convex spaces $Z \supset Y$, every compact operator $T : Y \to X$ has an extension to a compact operator $\overline{T} : Z \to X$. The results obtained depend strongly on the spherical completeness of the ground field. On the other hand, the situation here is completely different from its Archimedean counterpart. Our results also lead to some new characterizations of spherically complete fields and of discretely valued fields.

Introduction. In [**16**], the authors characterize the (real or complex) Banach spaces X having the following property:

$(*)$ For all Banach spaces $Z \supset Y$ every compact operator $T : Y \to X$ has an extension to a compact operator $\overline{T} : Z \to X$.

They prove that X has property $(*)$ if and only if it is an $\mathcal{L}_\infty$-space.

Our first goal was to study this property in the non-Archimedean case. It turned out that not only was the situation completely different here, but also that the answer was surprisingly simpler. We found, indeed, that when the ground field $\mathbf{K}$ is spherically complete, then every non-Archimedean Banach space X over $\mathbf{K}$ has property $(*)$, and if $\mathbf{K}$ is not spherically complete, no $X \neq \{0\}$ has this property.

We therefore decided to look at the problem in the much more general frame of non-Archimedean locally convex spaces and called the property CEP (Definition 2.1). Here the situation is more complicated. If $\mathbf{K}$ is not spherically complete, still there are no nontrivial locally convex spaces over $\mathbf{K}$ with the CEP (Section 3). On the other hand, if $\mathbf{K}$ is spherically complete, lots of spaces have the CEP but not all of them (Section 4), and the situation is still different in the special case when the valuation on $\mathbf{K}$ is discrete (Section 5).

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