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# ON THE SOLVABILITY OF TWO SIMULTANEOUS SYMMETRIC CUBIC DIOPHANTINE EQUATIONS WITH APPLICATIONS TO SEXTIC DIOPHANTINE EQUATIONS

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ABSTRACT. This paper provides a necessary and sufficient condition for the solvability of the simultaneous diophantine equations $C_1(x,y) = C_1(u,v)$ and $C_2(x,y) = C_2(u,v)$ where $C_i(x,y)$, $i = 1,2$, are arbitrary binary cubic forms. If the forms $C_i(x,y)$ have a common factor, we obtain the complete solution of these equations; otherwise, we obtain infinitely many solutions provided the condition of solvability is satisfied. The method has been used to solve some diophantine problems such as finding triads of cubes with equal sums and equal products, finding two arithmetic progressions of six terms each with equal products of terms, as well as for solving certain sextic diophantine equations of the type $f(x,y) = f(u,v)$.

This paper is concerned with the solvability of the simultaneous diophantine equations

$$C_1(x,y) = C_1(u,v), \tag{1.1}$$
$$C_2(x,y) = C_2(u,v), \tag{1.2}$$

where $C_i(x,y)$, $i = 1,2$, are two distinct binary cubic forms defined by

$$C_1(x,y) = a_0x^3 + a_1x^2y + a_2xy^2 + a_3y^3, \tag{1.3}$$
$$C_2(x,y) = b_0x^3 + b_1x^2y + b_2xy^2 + b_3y^3, \tag{1.4}$$

where the coefficients $a_j$, $b_j$, $j = 0,1,2,3$, are integers. As both equations (1.1) and (1.2) are homogeneous, any rational solution of these equations may be multiplied by a suitable constant to obtain a solution in integers. A solution in integers, say $(x_1, y_1, u_1, v_1)$, will be said to be primitive if $\gcd(x_1, y_1, u_1, v_1) = 1$. Further, any solution other than the trivial solution $x = u$, $y = v$ will be said to be nontrivial.

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