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THE STABLE SET OF ASSOCIATED PRIMES OF THE IDEAL OF A GRAPH

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ABSTRACT. Let G be a graph and let I be the edge ideal of G. We give a constructive method for determining primes associated to the powers of I. Brodmann showed that the sets of associated primes stabilize for large powers of I. Our construction will yield this stable set and an upper bound on where the stable set will occur.

1. Introduction. In this paper we will study the sets of prime ideals that are associated to the powers of the edge ideal of a graph. In [**1**], Brodmann showed that when R is a Noetherian ring and I is an ideal of R, the sets $\operatorname{Ass}(R/I^n)$ stabilize for large n. That is, there exists a positive integer N such that $\operatorname{Ass}(R/I^n) = \operatorname{Ass}(R/I^N)$ for all $n \geq N$. Although the sets $\operatorname{Ass}(R/I^n)$ have been studied extensively (see [**5**] for instance), little is known about where the stability occurs or about which primes are in the stable set. If the ideal is generated by a regular sequence, then it is shown in [**3**, 2.1] that $\operatorname{Ass}(R/I^n) = \operatorname{Min}(R/I)$ for all n. If the ring R is Gorenstein and if I is a strongly Cohen-Macaulay perfect ideal generated by a d-sequence, then in [**6**, Theorem 2.2] the stable set is described and an upper bound on where it occurs is given. However, if the generators of the ideal do not form a d-sequence, very little is known. Even for special classes of ideals such as monomial ideals or ideals defining simplicial complexes, the stable set is unknown.

In this paper we will work with a class of monomial ideals, the edge ideals of graphs. These are ideals whose generators are square-free monomials of degree two. We will give a construction that produces the primes that are in the stable set of the ideal of a graph and that gives an upper bound for where the stability occurs.

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