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AN ALTERNATIVE FOR THE SPECTRAL RADIUS OF POSITIVE INTEGRAL OPERATORS —A FUNCTIONAL ANALYTIC APPROACH

LUDWIG KOHAUPT

ABSTRACT. In a former paper, the author has investigated the solution of Fredholm integral equations of the second kind with positive integral operators in weighted function spaces. These results can be obtained more easily by using a functional analytic approach. We demonstrate this for an alternative theorem concerning the spectral radius of positive integral operators. To this end, first some refinements of results on positive operators in abstract Banach spaces have to be derived.

0. Introduction. In [**5**], among other things, alternative theorems for positive integral operators are derived in a closed subspace $C_{\sigma^{-1}}(\Omega) \subset C_{\sigma^{-1}}(\Omega_N)$, respectively in $L_\sigma(\Omega)$, where σ is a continuous weight function, σ^{-1} means $1/\sigma$ and where the weighted spaces fulfill the inclusions $C_{\sigma^{-1}}(\Omega) \subset C(\Omega)$ and $L_\sigma(\Omega) \supset L(\Omega)$ with $\Omega = [a,b]$. The cases are handled there separately.

In this paper we show that both cases can be treated in a unified manner by using a functional analytic approach and the duality relation $[L_\sigma(\Omega)]^* = L_{\infty,\sigma^{-1}}(\Omega)$ where $L_\sigma(\Omega)$, respectively $L_{\infty,\sigma^{-1}}(\Omega)$, is a generalization of $L(\Omega)$, respectively $L_\infty(\Omega)$.

The first three sections form the functional analytic part and the last section the application part.

The paper is structured as follows. In Section 1 some preliminaries and notations are given. Section 2 derives the relation $\rho(B) = \|B\|_\kappa$ under weaker conditions than known so far, where κ is a positive eigenvector of the positive operator B. In Section 3 an abstract alternative theorem for the spectral radius is proven. Finally, in Section 4 the alternative theorem is applied to integral operators. The conditions imposed on the integral kernel are usually fulfilled with

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