ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 30, Number 2, Summer 2000

ON THE LOCAL CAUCHY PROBLEM FOR HAMILTON JACOBI EQUATIONS WITH A FUNCTIONAL DEPENDENCE

ZDZISŁAW KAMONT

ABSTRACT. The Cauchy problem for a nonlinear functional differential equation is considered. A theorem on the existence of classical solutions defined on the Haar pyramid is proved. The method of differential inequalities is used.

Differential equations with a deviated argument and differential integral equations can be obtained by specializing given operators.

1. Introduction. We will denote by $C(X,Y)$ the class of all continuous functions from X into Y where X and Y are metric spaces. We will use vectorial inequalities with the understanding that the same inequalities hold between their corresponding components. For $y = (y_1, \dots, y_n) \in R^n$, we put $\|y\| = |y_1| + \dots + |y_n|$. Let E be the Haar pyramid

$$E = \{(x,y) \in R^{1+n} : x \in [0,a],\ -b + M\,x \le y \le b - M\,x\},$$

where b, $M \in R^n_+$, $R_+ = [0, +\infty)$, $b = (b_1, \dots, b_n)$, $M = (M_1, \dots, M_n)$ and $b - Ma > 0$. Write

$$E_0 = [-r_0, 0] \times [-b, b] \quad \text{where } r_0 \in R_+, \quad E^* = E_0 \cup E$$

and

$$E_x = E^* \cap ([-r_0, x] \times R^n), \quad \tilde{E}_x = E \cap ([0,x] \times R^n)$$

where $0 \le x \le a$. Suppose that $f : E \times C(E^*, R) \times R^n \to R$ and $\varphi : E_0 \to R$ are given functions. Consider the Cauchy problem

$$\partial_x z(x,y) = f(x, y, z, \nabla_y z(x,y)), \tag{1}$$

$$z(x,y) = \varphi(x,y) \quad \text{on } E_0, \tag{2}$$

Received by the editors on October 4, 1997, and in revised form on April 16, 1999.
AMS *Mathematics Subject Classifications.* 35R10, 35R45, 35F25.