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POLYNOMIALS ON SCHREIER'S SPACE

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ABSTRACT. We introduce a weakened version of the Dunford-Pettis property and give examples of Banach spaces with this property. In particular, we show that every closed subspace of Schreier's space S enjoys it. As an application we characterize the weak polynomial convergence of sequences, show that every closed subspace of S has the polynomial Dunford-Pettis property of Biström et al. and give other polynomial properties of S.

A subset $A = \{n_1 < \cdots < n_k\}$ of the natural numbers $\mathbf{N}$ is said to be *admissible* if $k \leq n_1$. Schreier's space S [**22**], [**4**] is the completion of the space c_{00} of all scalar sequences of finite support with respect to the norm:

$$\|x\|_S := \sup\left\{\sum_{j\in A} |x_j| : A \subset \mathbf{N} \text{ is admissible}\right\}, \quad \text{for } x = (x_j)_{j=1}^{\infty}.$$

Some basic properties of S may be seen in [**6**]. Schreier's space has been used to provide counterexamples in Banach space theory [**2**], [**6**], [**7**], [**20**], [**21**].

In this paper we introduce a weakened version of the Dunford-Pettis property and give examples of Banach spaces with this property. In particular, we show that every closed subspace of S enjoys it. It is well known that a reflexive Banach space with the Dunford-Pettis property must be finite dimensional. The same is true for a Banach space with the Banach-Saks property and the weak Dunford-Pettis property. As an application we investigate polynomial properties of S, characterizing the sequences which converge in the weak polynomial topology that we

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