

## THE STRUCTURE OF SYMMETRY GROUPS OF ALMOST PERFECT ONE FACTORIZATIONS

DELLA DUNCAN AND EDWIN IHRIG

**ABSTRACT.** The concept of  $j$ -perfection of a one factorization is introduced. A one factorization is perfect if and only if it is 2-perfect. We call 3-perfect one factorizations almost perfect one-factorizations (or APOFs). First we give some general results concerning the automorphism groups of  $j$ -perfect one factorizations, and then we classify all APOFs which have more than one automorphism of order two with fixed points. Several structure theorems for the automorphism groups of APOFs are also given.

**0. Introduction.** Let  $\Gamma$  be an undirected simple connected graph with vertex set  $V$ . A one factor (or perfect matching) of  $\Gamma$  is a subgraph of  $\Gamma$  in which each vertex in  $V$  has degree one. A one factorization  $\mathcal{F}$  of  $\Gamma$  is a collection of one factors of  $\Gamma$  so that each edge of  $\Gamma$  occurs in exactly one of the one factors in  $\mathcal{F}$ . Associated with any one factorization  $\mathcal{F}$  is its group of automorphisms,  $\text{Aut}(\mathcal{F})$ , which is the collection of all the permutations of  $V$  which transforms any one factor in  $\mathcal{F}$  in to another one factor in  $\mathcal{F}$ .

Given any finite group  $G$ , Cameron has shown there is a one factorization of  $K_{2n}$  (the complete graph on  $2n$  vertices) which has  $G$  as its automorphism group. This result is a consequence of an analogous result by Mendelson given for Steiner triple systems (see [7]). We provide the details here in Section 4 for easy reference. This result means that there are unlikely to be any strong structure results that apply to the automorphism groups of a general one factorization.

However, in contrast to this situation, it has been discovered that one factorizations with extra structure can have automorphism groups with a very restricted form. An example of this are perfect one factorizations. A perfect one factorization (POF) is a one factorization in which the union of any two distinct one factors is connected. There are a number of very restrictive theorems governing the nature of the automorphism groups of POFs. See [8] for an overview. For example,

---

Received by the editors on July 8, 1998, and in revised form on April 8, 1999.

Copyright ©2000 Rocky Mountain Mathematics Consortium