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LOCAL RIGIDITY OF HYPERSURFACES IN REAL EUCLIDEAN SPACES

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ABSTRACT. This paper studies the rigidity of isometric immersions of n-dimensional Riemannian manifolds into the $(n+1)$-dimensional Euclidean space. In the formalism of jet theory for partial differential equations, we investigate the relationship between the rigidity of immersions and the existence of complete systems of order 2. We define the notion of jet-rigidity as an intrinsic concept of a Riemannian structure and show that nondegenerate isometric immersions of jet-rigid Riemannian manifolds are actually rigid. We also get another proof of the classical Beez-Killing theorem on the rigidity of hypersurfaces.

0. Introduction and statement of the main results. Let (Ω, g) be an n-dimensional smooth (C^∞) manifold with smooth Riemannian metric g. A C^1 map f of Ω into $\mathbf{R}^{n+r}$ is an isometric immersion if and only if f satisfies

$$(1) \qquad \sum_{\alpha=1}^{n+r} \frac{\partial u^\alpha}{\partial x^i}\frac{\partial u^\alpha}{\partial x^j} = g_{ij}(x), \quad 1 \le i, j \le n,$$

where $x = (x^1, \dots, x^n)$ is a local coordinate system of Ω and $g_{ij}(x) = g((\partial/\partial x^i), (\partial/\partial x^j))$. We are concerned in this paper with the rigidity of solutions of (1). Since $g_{ij} = g_{ji}$, the number of equations in (1) is $n(n+1)/2$ and thus (1) is underdetermined if $r > n(n-1)/2$ and overdetermined if $r < n(n-1)/2$.

An isometric immersion $f : \Omega \to \mathbf{R}^{n+r}$ is said to be rigid if for any isometric immersion $\tilde{f}$ there exists an isometry Φ of $\mathbf{R}^{n+r}$ such that $\tilde{f} = \Phi \circ f$. We say an isometric immersion depends on constants if there is an integer k such that if f and $\tilde{f}$ are two isometric immersions

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