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# AN EXTENSION OF THE BOLSINOV-FOMENKO THEOREM ON ORBITAL CLASSIFICATION OF INTEGRABLE HAMILTONIAN SYSTEMS

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ABSTRACT. The main result of the paper is an extension of the Bolsinov-Fomenko theorem on topological orbital classification of nondegenerate integrable Hamiltonian systems with two degrees of freedom on three-dimensional constant energy manifolds (1994). Namely, it is shown that their restriction that the integral has no critical circles with nonorientable separatrix diagrams can be omitted. Our proof is based on an analogue of obstruction theory for certain types of Seifert fibrations.

**1. Introduction.** In 1994 Bolsinov and Fomenko [**3**] proved a theorem on topological orbital classification of nondegenerate integrable Hamiltonian systems with two degrees of freedom on three-dimensional constant energy manifolds. For motivation and a short survey see [**5**, Section 1], [**3**, Section 1]. They showed that two such systems are equivalent if certain invariants are. The invariant is a graph with a number of additional labels associated to its vertices and edges, Section 2. One of the restrictions they had to impose was that the Hamiltonian systems do not have unstable isolated periodic orbits with a nonorientable separatrix. Since the existence of such orbits is, in some sense, a generic property which appears in many classical integrable cases, e.g., in the Kovalevskaya top, it is desirable to remove this restriction. In this paper we show that the Bolsinov-Fomenko theorem is also true without this restriction.

**Theorem 1.1** (cf. [**3**, Theorem 4.1]). *Let $X$ be the set of nondegenerate integrable Hamiltonian systems with two degrees of freedom*

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