

## CURVES IN $\mathbf{P}^3$ WITH GOOD RESTRICTION OF THE TANGENT BUNDLE

GEORG HEIN

**ABSTRACT.** We extend the Shatz stratification of sheaves to arbitrary families of projective schemes. This allows a stratification of Hilbert schemes. We investigate how the Harder-Narasimhan polygon of the restriction of the tangent bundle  $\Theta_{\mathbf{P}^n}$  to space curves reflects the geometry of these curves and their embeddings.

**Introduction.** The stability of the tangent bundle  $\Theta_{\mathbf{P}^3}$  of the projective space does not yield the stability of its restriction to an arbitrary space curve. On the one hand we expect that for a general curve  $X$  in  $\mathbf{P}^3$  the restriction of  $\Theta_{\mathbf{P}^3}$  to the curve  $X$  is stable. On the other hand, we may ask what does the instability of the restriction imply for the geometry of the curve and its embedding? It is natural to consider this problem in the context of the Hilbert scheme  $\text{Hilb}(d, g)$  which parameterizes all space curves in  $\mathbf{P}^3$  of degree  $d$  and arithmetic genus  $g$ . To measure the instability we use a slight generalization of the well-known Harder-Narasimhan polygon. We obtain an upper semi-continuous map  $\text{HNP} : \text{Hilb}(d, g) \rightarrow \{\text{polygons}\}$  with finite image by assigning each curve  $[X]$  the Harder-Narasimhan polygon of  $\Theta_{\mathbf{P}^3|X}$ . We ask what is the image of the map HNP?

The first naturally arising subquestion is to decide if the stable polygon lies in the image. Under the assumption  $g \leq (4/3)d - 4$ , we show that this is indeed true. Furthermore, we derive good upper bounds for the map HNP. These bounds allow us to describe the image of HNP, for curves of degree less than or equal to six. Moreover, we are able to decide which special properties the curves in different strata have. Such special properties are low values of  $i$ -gonalities of a curve, secant lines of high order and being contained in a smooth quadric surface. To illustrate this we regard in Section 2.4 the Hilbert scheme  $\text{Hilb}(6, 3)$  which has two strata in the open set parameterizing smooth curves.

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