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FAITHFULNESS AND CANCELLATION OVER NOETHERIAN DOMAINS

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A recent theme in the study of commutative rings is to observe the necessary structure present in a ring in order for a particular theorem in abelian group theory to hold true when interpreted for the ring. A theorem of Warfield [**13**] asserts that when X is a nonzero subgroup of the group of rational integers, then $G \cong_{\mathrm{nat}} \mathrm{Hom}\,(X, XG)$ for any torsion-free $\mathrm{End}\,(X)$-module G, here XG has the appropriate meaning as a subgroup of the divisible hull of G. A module M over a ring R is called *faithful*, in the sense of Arnold and Lady [**2**], if $IM = M$ implies $I = R$ for any ideal I of R. In our work we will consider several variations of the faithful concept and relate them to Warfield's result.

Below, all unadorned Hom and $\otimes$ symbols are with respect to the integral domain R, and Q is used to represent the quotient field of R. Given a torsion-free module M over R, we define the *ring of fractions* of M as $E_M = \{t \in Q \mid tM \subseteq M\}$ which is the largest overring of R in Q over which M is a module. The rank of a torsion-free module M is the dimension of the Q-vector space, $Q \otimes M$. In case X is a torsion-free module of rank one, E_X is just the endomorphism ring of X. We note without future reference that if S is an overring of R in Q and X and Y are torsion-free modules with $S \subseteq E_X \cap E_Y$, then $\mathrm{Hom}\,(X, Y) = \mathrm{Hom}_S(X, Y)$. In particular, E_X is independent of the base ring R. When P is a prime ideal of R, the notation S_P represents the localization of the module S at P and is therefore $S_P = R_P \cdot S \subseteq Q$.

1. Noetherian domains.

Definitions. An integral domain R is called *strongly faithful* if, for any submodule X of Q and torsion-free module G, one has $G \cong_{\mathrm{nat}} \mathrm{Hom}\,(X, XG)$ when $E_X \subseteq E_G$. Here XG is the submodule of the divisible hull of G, QG, generated by xg such that $x \in X$ and $g \in G$.

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