

## SUPPLENESS OF SOME QUOTIENT SPACES OF MICROFUNCTIONS

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ABSTRACT. The theory of microfunctions which Sato introduced contributed very much to that of linear partial differential equations. He constructed a useful transformation of these equations by expanding the analytic singularities of a hyperfunction onto the cosphere bundle. We stick not only to the analytic singularities but also to some other singularities of some subclasses of hyperfunctions which Komatsu [7], Hörmander [4], Eida [2] and others have introduced. For this reason, in this paper we develop the theory of some quotient spaces of microfunctions. We state the suppleness of the sheaves of these functions which is important for the theory of microdifferential equations. Our investigations enable us to introduce the notions  $SS^*$  and  $SS^{1/*}$  for hyperfunctions.

**1. Notations and definitions.** We consider some classes of hyperfunctions, that is, ultradifferentiable functions and ultradistributions. We will recall the basic definitions [2, 6]. Let  $M_p$  be a sequence of positive numbers satisfying the following conditions.

$$\begin{aligned} \text{(M.0)} \quad & M_0 = M_1 = 1; \\ \text{(M.1)} \quad & M_p^2 \leq M_{p-1}M_{p+1}, \quad p = 1, 2, \dots; \\ \text{(M.2)} \quad & M_p/M_q M_{p-q} \leq AB^p, \quad 0 \leq q \leq p; \\ \text{(M.3)'} \quad & \sum_{p=1}^{\infty} M_{p-1}/M_p < \infty. \end{aligned}$$

We note that the Gevrey sequence  $M_p = (p!)^s$  or  $p^{ps}$  or  $\Gamma(1 + ps)$ , for  $s > 1$ , satisfies the above conditions.

A function  $f(x)$  on an open set  $U$  in  $\mathbf{R}^n$  is called an ultradifferentiable function of class  $(M_p)$  (respectively  $\{M_p\}$ ), if for any compact set  $K$

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Received by the editors on June 17, 1999, and in revised form on September 8, 1999.

1991 AMS *Mathematics Subject Classification.* 32A40, 46F15.