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AN EXPLICIT ZERO-FREE REGION FOR THE RIEMANN ZETA-FUNCTION

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ABSTRACT. This paper gives an explicit zero-free region for the Riemann zeta-function derived from the Vinogradov-Korobov method. We prove that the Riemann zeta-function does not vanish in the region $\sigma \geq 1 - .00105 \log^{-2/3} |t| (\log\log |t|)^{-1/3}$ and $|t| \geq 3$.

1. Introduction. It is now well known that the problem involving prime numbers can be related to the study of the Riemann zeta-function. In 1860, Riemann in [**17**] showed that the key to the deeper investigation of the distribution of the primes lies in the study of the function which is now called the Riemann zeta-function. Let $s = \sigma + it$ be a complex variable. For $\sigma > 1$, the Riemann zeta-function is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}. \tag{1}$$

The above series converges absolutely and uniformly on the half plane $\sigma \geq \sigma_0$ for any $\sigma_0 > 1$. It can be extended to be a regular function on the whole complex plane $\mathbf{C}$, except at $s = 1$, which is the only pole of the Riemann zeta-function and at which the function has residue 1. The general definition of the Riemann zeta-function may be referred to by its functional equation. That is,

$$\pi^{-s/2}\Gamma(s/2)\zeta(s) = \pi^{-(1-s)/2}\Gamma((1-s)/2)\zeta(1-s). \tag{2}$$

Here Γ is the factorial function of a complex variable and $\Gamma(n) = (n-1)!$ for every positive integer n. The pole of Γ at $s = 0$ corresponds to that of $\zeta(s)$ at $s = 1$. The other poles of Γ at $s = -n$ for positive integers

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