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MODULES OVER DOMAINS LARGE IN A COMPLETE DISCRETE VALUATION RING

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ABSTRACT. We consider a class of domains R containing a maximal ideal $\mathfrak{N}$ such that R is not complete with respect to the $\mathfrak{N}$-adic topology, but $T = R_{\mathfrak{N}}$ is a complete DVR. Such domains are called T-large because of the way to construct them. We characterize a T-large domain R to be of the form $R = T \cap V$, where V is a mildly restricted valuation domain of Q, the field of fractions of T. We show that the completion $\hat{V}$ of V has infinite rank as a V-module. We investigate finite rank torsion-free modules M over a T-large domain R which are Hausdorff in the $\mathfrak{N}$-adic topology. Making use of known results on V-modules, we obtain the following results: there exist indecomposable torsion-free Hausdorff R-modules of any fixed rank n; every cotorsion-free Hausdorff R-algebra of rank n is the endomorphism algebra of a torsion-free module of rank $3n$; the Krull-Schmidt theorem fails, that is, there exist finite rank torsion-free Hausdorff R-modules which admit non-isomorphic decompositions into indecomposable summands.

Introduction. In his 1962 book [**6**], Nagata exhibited the first example of a noncomplete discrete valuation ring R such that $[\hat{Q} : Q] < \infty$, where $Q, \hat{Q}$ are the field of fractions of R and its completion $\hat{R}$, respectively. The DVR's satisfying this property were called Nagata valuation domains in [**9**].

Recently the second author [**9**] and Arnold and Dugas [**1**] investigated torsion-free modules of finite rank over Nagata valuation domains R. In particular, in [**9**] it was proved that if $[\hat{Q} : Q] = 2$, then every finite rank torsion-free indecomposable R-module has rank ≤ 2; in [**1**] it is shown that $[\hat{Q} : Q] = 3$ implies that every finite rank torsion-free indecomposable R-module has rank ≤ 3, while if $[\hat{Q} : Q] \geq 4$, then there exist finite rank torsion-free indecomposable R-modules of arbitrarily large rank. It is worth noting that the Krull-Schmidt theorem holds for finite rank torsion-free modules over Nagata valuation domains since

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