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NORMALITY OF COMPLEX CONTACT MANIFOLDS

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ABSTRACT. Complex contact metric manifolds are studied. Normality is defined for these manifolds and equivalent conditions are given in terms of ∇G and ∇H. GH-sectional curvature and $\mathcal{H}$-homothetic deformations are defined. Examples of normal complex contact metric manifolds with constant GH-sectional curvature c are given for $c \geq -3$.

1. Introduction. The theory of complex contact manifolds started with the papers of Kobayashi [**12**] and Boothby [**4**], [**5**] in late 1950's and early 1960's, shortly after the celebrated Boothby-Wang fibration in real contact geometry [**6**]. It did not receive as much attention as the theory of real contact geometry. In 1965, Wolf studied homogeneous complex contact manifolds [**17**]. Recently, more examples are appearing in the literature, especially twistor spaces over quaternionic Kähler manifolds (e.g., [**13**], [**14**], [**15**], [**16**], [**18**]). Other examples include the odd dimensional complex projective spaces [**9**] and the complex Heisenberg group [**1**].

In the 1970's and early 1980's there was a development of the Riemannian theory of complex contact manifolds by Ishihara and Konishi [**8**], [**9**], [**10**]. However, their notion of normality as it appears in [**9**] seems too strong since it does not include the complex Heisenberg group and it forces the structure to be Kähler. In this paper we introduce a slightly different notion of normality which includes the complex Heisenberg group.

In Section 2 we give the necessary definitions and some basic facts about complex contact metric manifolds. In Section 3 we define normality and give the theorem which states the necessary and sufficient conditions, in terms of the covariant derivatives of the structure tensors, for a complex contact metric manifold to be normal. We discuss some curvature properties of normal complex contact metric manifolds in Section 4.

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