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CHARACTERIZING A CLASS OF WARFIELD MODULES BY RELATION ARRAYS

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Introduction. In this paper we examine the Warfield modules in a class $\mathcal{H}$ with the property that the torsion submodule is a direct sum of cyclics and the quotient modulo torsion is divisible of arbitrary rank. We give necessary and sufficient conditions about when modules in $\mathcal{H}$ are Warfield if their torsion-free rank is countable and the indicators of torsion-free elements are exclusively of ω-type or exclusively of finite-type. We give two examples to show that the conditions placed on such modules cannot be eliminated. Indeed, we explicitly describe two non-Warfield modules in the class $\mathcal{H}$ of torsion-free rank 2 where the indicators of all torsion-free elements are either of finite-type or of ω-type, respectively, but yet the modules do not satisfy the aforementioned conditions. In addition we prove that a Warfield module is equivalent to a simply presented module if the indicators of torsion-free elements are all of ω-type or all of finite-type. We show that this result is in some sense the best possible by giving an example of a Warfield module in $\mathcal{H}$ which is not simply presented whose torsion-free rank is 2 and contains indicators of both the finite and ω-type. This example complements one given by Warfield of a mixed module of torsion-free rank 1. The proofs of our results rely on the description of these modules by generators and relations, their corresponding relation arrays, and the results established in [**3**], [**4**], [**5**], [**6**].

1. Notation. Let $\mathbf{N}$ denote the set of natural numbers and $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. Let R denote a *discrete valuation domain*, i.e., a local principal ideal domain with prime p and quotient field $\mathbf{F}$. All modules are understood to be R-modules.

We now recall from [**5**] the definition of a module by generators and relations. Let G be a module in the class $\mathcal{H}$ of torsion-free rank d

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