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SECOND-ORDER DIFFERENTIAL OPERATORS WITH INTEGRAL BOUNDARY CONDITIONS AND GENERATION OF ANALYTIC SEMIGROUPS

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ABSTRACT. Consider a second-order differential operator $Lu = u'' + q_1(x)u' + q_0(x)u$ with integral boundary conditions of the form

$$\int_a^b R_i(t)u(t)\,dt + \int_a^b S_i(t)u'(t)\,dt = 0, \quad i = 1, 2.$$

We study sufficient conditions on the functions R_i and S_i, $i = 1, 2$, such that the operator L is the generator of an analytic semigroup of operators on $L^p(a, b)$. The generation of analytic semigroups is proved by showing the estimate

$$\|R(\lambda : L)\| \leq \frac{M}{|\lambda|}$$

for the resolvent operator in a suitable sector of the complex plane. The motivation for this work is to generalize the results in [**3**], where nonseparated boundary conditions were considered.

1. Introduction. We consider a second-order differential operator of the form

$$l(u) = u'' + q_1(x)u' + q_0(x)u, \quad x \in (a, b), \tag{1.1}$$

where each $q_i(x)$ is a regular function with complex values. We can associate to $l(u)$ a variety of boundary conditions, in particular, the *nonseparated* ones:

$$\begin{cases} a_1u(a) + b_1u'(a) + c_1u(b) + d_1u'(b) = 0, \\ a_2u(a) + b_2u'(a) + c_2u(b) + d_2u'(b) = 0. \end{cases} \tag{1.2}$$

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