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NORM CONVERGENCE OF MOVING AVERAGES FOR τ-INTEGRABLE OPERATORS

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ABSTRACT. It is shown that if α is a positive linear map on $L^1(M,\tau)$ of a von Neumann algebra M with a faithful normal (semi-)finite trace τ which is norm-reducing for both the operator norm and the integral norm associated with τ, then the moving averages converge in L^p-norm, $1 \le p < \infty$. Using this result it has been shown that similar norm convergence results hold for some super-additive processes in $L^p(M,\tau)$ relative to τ-preserving α.

1. Introduction. This article concerns some strong convergence results for moving averages in the von Neumann algebra setting. Beginning with the celebrated theorem of Lance and Yeadon, there has been great interest in extending various results in classical ergodic theory into operator algebras, particularly to von Neumann algebras. For a review, see [**6**], [**8**]. Recently, such activities have been revived in the context of obtaining various weighted ergodic theorems in von Neumann algebras [**7**], [**9**], [**10**]. Study of convergence of moving averages in von Neumann algebra settings is new. Actually we will obtain norm convergence of moving averages for both additive and superadditive processes in a von Neumann algebra.

Let M be a von Neumann algebra with the unit I, and let τ be a faithful normal semi-finite trace on M. For the definition of L^p-spaces, $1 \le p \le \infty$, associated with (M,τ), see [**14**], [**12**], [**15**], [**4**]. $L^p = L^p(M,\tau)$, being noncommutative generalizations of the classical L^p-spaces, inherit most of their important properties. For example, the following form of the Hölder inequality holds [**4**]

$$\|xy\|_r \le \|x\|_p \|y\|_q$$

whenever $p,q,r>0$ and $p^{-1}+q^{-1}=r^{-1}$. When τ is finite, this implies that if $p>q$ then $L^p \subset L^q$.

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